Study of ^{3}He(e,e') longitudinal response functions with the integral-transform method

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The method of integral transforms is first applied for studying the ^{3}He longitudinal response functions. The transforms are calculated from localized bound-state-type solutions to an inhomogenous Schrödinger-type three-body equation. The multipole expansion of the solutions is considered. The whole calculation is checked using a sum-rule test. Several versions of local s-wave spin-dependent potentials supplemented with a singlet p-wave potential and with the proton-proton Coulomb interaction are used as a two-nucleon input. The conventional charge density operator involving free-nucleon form factors is utilized. The three-body equations are solved with the help of the hyperspherical expansion and a complete convergence is achieved. The final-state interaction is thus fully taken into account. The $q = 300$ MeV/c and 500 MeV/c values are considered. It is found that the contribution of the $T = 3/2$ final states to the problem is suppressed and it amounts about 15%. This might be ascribed to symmetry requirements. The contributions of the p-wave NN interaction and of the Coulomb interaction are found to amount several per cent. Uncertainty due to different choices of s-wave NN forces is of a similar magnitude provided that the low-energy NN data are properly described. The results are compared with the integral transforms of the experimental response functions. For $q = 300$ MeV/c experimental and theoretical results coincide within their uncertainties. For $q = 500$ MeV/c a noticeable difference is detected.

Many calculations testify to the fact that the conventional form of the nuclear charge density is inapplicable for description of the elastic form factors of three- and four-nucleon nuclei at $q > 2.5$ fm^{-1} values. However the elastic scattering occurs with a quite low probability at such q values and some non-typical configurations including those where all the nucleons are close together may contribute substantially. In this connection it seems important to test a form of the nuclear 4-current in the inelastic processes and to study the (e, e') response functions. This requires a proper account of the nuclear final-state interaction.

We study (e, e') response functions

$$R(q,\omega) = \bar{\sum}_{M_0} \int df | < \psi_f | \hat{O} | \psi_0 > |^2 \delta(E_f - E_0 - \epsilon) \qquad (1)$$

where ϵ is the nuclear excitation energy. Direct evaluation of R with a full account of the final-state interaction is quite complicated even in the three-body case. Indeed, it requires obtaining the whole set of the final-state continuum wave functions ψ_f at the same energy and summing their contributions. As far as we know only one such an investigation was performed in the literature [1]. A local s-wave central potential [2] was utilized there while interaction in higher partial waves and Coulomb interaction were disregarded.

In this paper we present the first microscopical study of the ^{3}He longitudinal response functions R_l with the help of the method of integral transforms [3-6]. While the final-state interaction is fully taken into account in our approach we avoid calculating the continuum-spectrum wave functions ψ_f. We explore an ability of the method in an A=3 problem. We study sensitivity of the results to a choice of NN force and we clarify a role of an interaction in higher partial waves and that of the Coulomb interaction. We compare our results with experiment [7,8] at $q = 300$ and 500 MeV/c.

Define the reduced transition operator and the response function

$$\tilde{O} = [\tilde{G}_E^p(Q)]^{-1} \hat{O}, \qquad \tilde{R}(q,\omega) = [\tilde{G}_E^p(Q)]^{-2} R(q,\omega).$$

Here $Q^2 = q^2 - \omega^2$ and [9] $\tilde{G}_E^p(Q) = [1 + Q^2/(4M^2)]^{-1/2} G_E^p(Q)$ where G_E^p is the proton Sachs form factor. We calculate the integral transform of the response [3,10]

$$\Phi(q,\sigma) = \int_{\epsilon_{min}}^{\infty} (\sigma+\epsilon)^{-1} \tilde{R}(q,\omega) d\epsilon \tag{2}$$

instead of the response itself. We use the conventional single-nucleon expression for the charge density $\hat{O}$. Then to a very good approximation one can disregard the ω-dependence of the $\tilde{O}$ operator and use the expression

$$\tilde{O}(\mathbf{q}) = \sum_{j=1}^{A} [\frac{1-\tau_{zj}}{2} + \frac{G_E^n(q)}{G_E^p(q)} \frac{1+\tau_{zj}}{2}] e^{i\mathbf{q}\mathbf{r}'_j} \tag{3}$$

where $\mathbf{r}'_j = \mathbf{r}_j - \mathbf{R}_{c.m}$. It has been shown [3,10] that $\Phi(q,\sigma)$ can be calculated by first solving for the *localized* solution to the following inhomogenous equation

$$(H - E_0 + \sigma)\tilde{\Psi} = \tilde{O}\psi_0 \tag{4}$$

where as in Eq. (1) ψ_0 is the ground-state wave function and E_0 is the ground-state energy. In terms of $\tilde{\Psi}$ we have [3,10]

$$\Phi(q,\sigma) = < \tilde{\Psi} | \tilde{O}\psi_0 > - \sigma^{-1} \tilde{R}_{el} \tag{5}$$

where $\tilde{R}_{el}$ is the elastic contribution to the response.

The solution to Eq. (4) is much easier to obtain than the functions ψ_f entering Eq. (1). Indeed, in contrast to the latter functions there is no need to impose the complicated large-distance boundary conditions in order to fix a solution. The only condition of vanishing of the solution at large distances suffices. Therefore methods that are used in solving bound-state problems can be utilized here. Below we use real σ values in Eqs. (2), (4).

We have two possible ways to connect our theoretical calculations with experiment. One way [10] is to compare $\Phi(q,\sigma)$ with the same quantity obtained from the experimental $\tilde{R}(q,\epsilon)$ using Eq. (2). Another way [3] is to consider Eq. (2) as the integral equation and invert to obtain theoretical $\tilde{R}(q,\omega)$ and then compare the responses themselves. In the present work we use the first approach. This provides one only with a limited information since some R features are smeared out at performing the transform (2). Nevertheless, this still allows testing the $\hat{O}$ transition operator in Eq. (1). If, for example, the nucleon form factors

should be modified inside the nucleus this would produce a change in $\Phi(q,\sigma)$ of a magnitude comparable with the change in $(G_E^p)^2$.

Concerning the inversion problem, another version of the method [6] better suits for this purpose. The many-body equation to be solved in this version is similar to Eq. (4) yet. The only difference is that complex σ values are utilized. It is shown below that the left-hand side of Eq. (2) can be obtained from Eq. (4) with high accuracy and we achieved comparable accuracy for the complex σ values as well.

The calculations are performed at $q = 300$ and 500 MeV/c and they are compared with the Saclay [7] and Bates [8] experimental results. In this first calculation we use four versions of effective central local s-wave spin-dependent NN potentials [2,11,12] supplemented with a realistic singlet p-wave NN potential [13] and with the proton-proton Coulomb interaction, Only the components of latter interaction which are diagonal in the isospin T =1/2, 3/2 quantum numbers are retained in the calculation. Even these components prove to contribute little to the results, see below.

Under these assumptions on the nuclear dynamics, Eq. (4) is split into independent sets of equations with a given orbital momentum L, and isospin T of the system. It is convenient to calculate the right-hand sides of these equations in the following way. Since ψ_0 has $L = 0$ then only the components $\sim \sum_{M_L} Y_{LM_L}(\hat{\mathbf{r}}'_j)Y^*_{LM_L}(\hat{\mathbf{q}})$ from the expansion of $\exp(i\mathbf{qr}'_j)$ from Eq. (3) contribute to the problem for a given L value. R is independent of a $\mathbf{q}$ direction due to averaging over M_0 in Eq. (1). Let $\mathbf{q}$ be directed along the z axis. Then only the components with $M_L = 0$ give non-zero contributions and hence only the components of $\tilde{\Psi}$ with $M_L = 0$ are different from zero. We have

$$(H - E_0 + \sigma)\tilde{\Psi}_{LT} = \tilde{O}_{LT}\psi_0, \tag{6}$$

$$\tilde{O}_{LT} = \sum_{M_T} |TM_T >< TM_T| \sum_{j=1}^{3} [\frac{1-\tau_{zj}}{2} + \frac{G_E^n(q)}{G_E^p(q)}\frac{1+\tau_{zj}}{2}] j_L(qr_j)Y_{L0}(\hat{\mathbf{r}}'_j), \tag{7}$$

$$\Phi_{LT}(q,\sigma) =< \tilde{\Psi}_{LT}|\tilde{O}_{LT}\psi_0 > -\delta_{L0}\sigma^{-1}R_{el}, \tag{8}$$

$$\Phi(q,\sigma) = 4\pi \sum_{L=0}^{\infty}(2L+1) \sum_{T=1/2,\,3/2} \Phi_{LT}(q,\sigma). \tag{9}$$

The functions $\tilde{\Psi}_{LT}$ have the same spin $S = 1/2$ as ψ_0.

We solve Eqs. (6) using the hyperspherical expansion. Eqs. (6) turn into sets of algebraic equations. Denote K the hyperspherical momentum and $[f]$ the type of symmetry of the spatial components of the basis functions. Our basis functions are of the form

$$R_n(\rho)\Gamma^{[f]\mu_f}_{KLM_LSM_STM_T;i}(\Omega, \sigma_{zj}, \tau_{zj}). \tag{10}$$

Here ρ is the hyperradial variable and Ω are hyperangular variables. The index i enumerates the functions with the same other quantum numbers. We use the Laguerre-type hyperradial basis functions [14]. We have developed a computer code to construct complete sets of the Γ basis functions from Eq. (10) with arbitrary quantum numbers using the the Raynal-Revai transformation [15]. The coefficients of this transformation are evaluated using the recurrent formula of the $K \to K + 2$ type [16].

The L values up to $L_{max} = 8$ are retained which provides convergence in Eq. (9). The net number of the basis functions with the same K, L, S, T and $[f]$ values grows linearly with K which in general leads to systems of linear equations of high dimensions. But in fact these dimensions can be considerably reduced. For this purpose we use the fact that nuclear forces we utilize here only act in the two NN orbital states. Namely, we specify orthonormalized basis functions in such a way [17] that only for two functions from each set of functions with the same K, L, S, T and $[f]$ the matrix elements of NN force are different from zero. Only such functions are coupled together in Eqs. (6). Other functions do not include s or p components for each nucleon pair. They lead to decoupled equations in Eqs. (6) and they correspond to free-motion final states.

We retained the basis functions (10) with $K \leq K_{max} = 30$ in our calculations. This provides practical convergence of the results. The convergence trends are illustrated in Fig. 1 where dependence on K_{max} of the calculated $\Phi_{L=2,T=1/2}(q, \sigma)$ contribution is shown at $q = 300$ MeV/c and $\sigma = 1$ MeV as an example. In order to check stability of our results with respect to K_{max} we performed their extrapolations to $K_{max} = \infty$. This was done using three-parameter fits of the form $\Phi(K_{max}) = \Phi(\infty) - c/K^{\gamma}_{max}$. Extrapolated values $\Phi(\infty)$

differ from $\Phi(K_{max} = 30)$ less than by 1%. Convergence with respect to hyperradial basis functions from Eq. (10) is achieved, too. The maximum number of coupled basis states equaled 861 at solving Eqs. (6). The calculated transforms are smooth functions similar to shown in Fig. 1a from Ref. 4 for the deutron case.

We make a comment concerning calculation of Φ at small σ values. It is necessary to avoid large cancellations in the right-hand side of Eq. (5). This is achieved if one uses in Eq. (6) for $L = 0$, $T = 1/2$ the same K_{max} value as that at calculating ψ_0. Then the pole terms cancel exactly.

There exists a test which enables us to check the calculation as a whole. Namely, the leading term of $\Phi(q, \sigma)$ at high σ values behaves as σ^{-1}. This term can be calculated independently using the sum rule,

$$\lim_{\sigma\to\infty} \sigma\Phi(q,\sigma) = \int_{\epsilon_{min}}^{\infty} \tilde{R}(q,\omega)d\epsilon = \bar{\sum}_{M_0} < \psi_0|\tilde{O}^{\dagger}\tilde{O}|\psi_0 > -\tilde{R}_{el}. \qquad (11)$$

This allows one to check the right-hand side of Eq. (9). Besides checking the calculation the test allows one to verify whether at high σ values the results are stable against increasing K_{max} and L_{max}.[1]

Various contributions to Eq. (9) are just the integral transforms of the corresponding contributions to R. We note that the $T = 3/2$ contributions prove to be suppressed in comparison with the $T = 1/2$ ones. The net relative $T = 3/2$ contribution to $\Phi(q, \sigma)$ ranges between 12 and 17% for all the σ values and the two q values considered. The contributions to the initial responses R should be of a similar magnitude. The reason for the suppression may be in that the spatial component of the final-state wave function which are symmetrical

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[1]This test, however, is not applicable for checking the matrix elements of the Hamiltonian in Eqs. (6). (When σ tends to infinity these matrix elements can be neglected.) In a preliminary version of this work [18] a considerable disagreement with experiment was found. This was due to an algebraic error. For mixed symmetry states the matrix elements of kinetic energy were erroneously twice as large as correct ones.

with respect to particle interchanges are present at $T = 1/2$ only. These components may provide more internucleon attraction than those of the other symmetries which increases amplitudes of the final-state wave functions inside the reaction zone.

We estimated an influence on the results of NN interactions in higher partial waves and of the proton Coulomb interaction. The results are shown in Fig. 2 for $q = 300$ MeV/c. We performed the calculation for the MT(I+III) potential supplemented with the realistic p-wave singlet interaction from Ref. 13 and with the Coulomb interaction. Then we switched off the p-wave interaction. Curve 1 shows the relative change in the results. The contribution from the triplet p-wave force is believed to be of the same size. After that we switched off the Coulomb interaction in addition. This produces Curve 2. It is seen that the contributions of the p-wave interaction and of the Coulomb interaction do not exceed several per cent or standard uncertainties in Φ functions obtained from experimental data. For $q = 500$ MeV/c these contributions are even much smaller.

Furthermore, we studied dependence of the results on the choice of the NN force. Central local s-wave NN forces utilized in our calculation include the MT(I+III) [2], S2,S3 [11] and EH [12] potentials. The MT(I+III) and S2 potentials reproduce the NN low-energy properties and s-wave NN phases up to high energies. The S3 potential fits the low-energy data and yields nearly correct values for the binding energies and rms radii of ^{3}He and ^{4}He. The EH potential fits the s-wave NN phases up to high energies but it does not reproduce properly the low-energy NN data (see Ref. 11). As above all these forces were supplemented with the realistic p-wave singlet interaction [13] and with the proton Coulomb interaction in our calculations. Curve 3 in Fig. 2 represents the relative difference in Φ between S2 and MT(I+III) potentials. In case of S3 and MT(I+III) potentials such a difference is approximately of the same value and of opposite sign. These differences do not exceed several per cent. Curve 4 represents such a difference between EH and MT(I+III) potentials. The latter difference exceeds ten per cent at small σ values which corresponds to substantial differences in low-energy parts of the responses. This can be attributed to the above-mentioned drawback of the EH potential. For $q = 500$ MeV/c differences in the

results for different potentials become considerably smaller.

Before comparing our calculation with experiment we make some comments on obtaining the integral transforms, Eq. (2)), of the experimental responses. In order to perform the integration in Eq. (2) with a sufficient accuracy and in particular to estimate the contribution from the unavailable high-ϵ tails of the spectra we approximate the experimental $\tilde{R}$ functions by the following analytical expressions: $a(\omega-\omega_{thresh})^{1/2}$ in the low ω region $\omega_{thresh} \leq \omega \leq \omega_1$, then $\sum_{n=0}^{N_b} b_n \omega^n$ in the region of the peak $\omega_1 \leq \omega \leq \omega_2$ and $\sum_{n=0}^{N_c} c_n (\omega_2/\omega)^{\alpha+n}$ in the region beyond the peak. The parameters a, b_n, c_n, α and ω_2 are chosen from the least-square procedure with additional requirements imposed of continuity of the fitting spectra and their first derivatives at ω_1 and ω_2 points. It turns out that a good description at quite wide ranges $\omega_2 \leq \omega \leq \omega_{max}$ of the spectra beyond the peaks are provided with a single term [19] $\sim \omega^{-\alpha}$. (In case, say, exponentially decreasing tail-terms the description is worse.) The best α values range between about 4 and 5 for all the q values considered. Similar α values were found [20] in the ^{4}He case. We extrapolate the fitted spectra beyond the ω_{max} values in order to take into account the contributions from the unavailable tails. In case of data from Ref. [7] these contributions proved to be quite small. They reach their maxima at high σ values where they are between 1 and 2%. In case of q=500 MeV data from Ref. [8] we did not succeed in producing stable extrapolations.

In Fig. 3 we compare the calculated Φ values and the values deduced from experiment. The theoretical calculation was done MT(I+III) NN potential [2] supplemented as above with p-wave singlet NN interaction and with the proton Coulomb interaction. The relative differences between the theoretical and experimental Φ values are shown. Taking into account 5% systematic uncertainties of the experimental data and also above-considered uncertainties of theoretical calculations one can say that that there is no significant difference between experiment and theory at q = 300 MeV/c. The deviation at q =500 MeV/c may be considered as significant. Detectable differences betweem experiment and theory at such q values for another set [8] of data were also obtained in Ref. [1] where the responses were calculated directly. They may be attributed to relativistic effects.

In conclusion, we applied the method of integral transforms for studying ^{3}He longitudinal response functions. This requires solving for a localized solution to an inhomogenous Schrödinger-type three-body equation. We elaborated techniques for this purpose and we obtained accurate solutions using central local NN potentials and the conventional expression for the nuclear charge density. We found that NN interaction in higher patial waves and the proton Coulomb interaction play a minor role in the problem. Uncertainties in the s-wave NN force proved to be not substantial as well provided that the low-energy NN-data are properly described. We compared the calculated integral transforms with experiment and we found that at q =300 MeV/c they agree with each other within their uncertainties. For q =500 MeV/c noticeable deviations are found. The results obtained make it possible to solve the corresponding problem in the quite important α-particle case, in particular. In addition, we found that the final-state $T = 3/2$ contributions to the problem are suppressed.

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References

[1] E. van Meijgaard and J. A. Tjon, Phys. Rev. C **45**, 1463 (1992); Phys Lett **B228**, 307 (1989)

[2] R. A. Malfliet and J. A. Tjon, Nucl. Phys. **A127**, 161 (1969).

[3] V. D. Éfros, Yad. Fiz. **41**, 1498 (1985) [Sov. J. Nucl. Phys. **41**, 949 (1985)].

[4] V. D. Efros, W. Leidemann and G. Orlandini, Few-Body Syst., **14**, 151 (1993).

[5] V. D. Éfros, Yad. Fiz. **56**, No. 7, 22 (1993) [Phys. At. Nucl. **56**, 869 (1993)] (*Proc. Int. Woorkshop*)

[6] V. D. Efros, W. Leidemann and G. Orlandini, Phys. Lett. B., in press.

[7] C. Marchand et al., Phys. Lett. **153B**, 29 (1985) and private communication.

[8] K. Dow et al. Phys. Rev. Lett. **61**, 1706 (1988); K. Dow, Ph.D. thesis, MIT, 1987.

[9] J. L. Friar, Phys. Lett. **43B**, 108 (1973); D. R. Yennie, M. Levy and D. G. Ravenhall, Rev. Mod. Phys. **29**, 144 (1957).

[10] V. D. Éfros, Ukr. Fiz. Zh. **25**, 907 (1980) [Ukr. Phys. J.] (sect. 2C).

[11] I. R. Afnan and Y. C. Tang, Phys. Rev. **175**, 1337 (1968).

[12] H. Eikemeier and H. H. Hackenbroich, Z. Phys. **195**, 412 (1966).

[13] R. de Tourreil, B. Rouben and D. W. L. Sprung, Nucl. Phys. **A242**, 445 (1973).

[14] G. Erens, J.L. Visschers and R. van Wageningen, Ann. Phys. **67**, 461 (1971).

[15] J. Raynal and J. Revai, Nuovo Cim. **A68**, 612 (1970).

[16] Ya. A. Smorodinskii and V. D. Éfros, Yad. Fiz. **17**, 210 (1973) [Sov. J. Nucl. Phys. **17**, 107 (1973)].

[17] V. D. Éfros, Yad. Fiz. **15**, 226 (1972) [Sov. J. Nucl. Phys. **15**, 128 (1972)].

[18] V. Yu. Dobretsov, V. D. Efros and Bin Shao, Kurchatov preprint IAE-5650/2, 1993.

[19] G. Orlandini, M. Traini, Phys. Rev. **C31**, 280 (1985).

[20] A. Yu. Buki et al., Kurchatov preprint IAE-5397/2, 1991; V. D. Efros, Few-Body Syst. Suppl. 6, 506 (1992).

CAPTION TO FIGURES

FIG. 1. Dependence of $\Phi_{LT}(q,\sigma)$ on K_{max} at $L = 2$, $T = 1/3$, $q = 300$ MeV/c and $\sigma = 1$ MeV.

FIG. 2. Relative differences in $\Phi(q,\sigma)$ between MT(I+III) potential [2] supplemented with p-wave singlet interaction plus Coulomb proton interaction and other NN inputs. Curve 1 - p-wave interaction switched off. Curve 2 - Coulomb interaction switched off in addition. Curve 3 - a difference with the S2 NN force [11]. Curve 4 - a difference with EH NN force [12]. (In the last two cases p-wave singlet interaction and Coulomb interaction are included in the calculation.)

Fig.3. Comparison of the calculated transforms $\Phi_{th}(q,\sigma)$ with experiment. Curve 1 is for experimental data from Ref. [7] for $q = 300$ MeV/c. Curve 2 is for experimental data from Ref. [8] for $q = 300$ MeV/c. Curve 3 is for experimental data from Ref. [8] for $q = 500$ MeV/c.

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